Law of Sines, Law of Cosines, and Area Formulas

Law of Sines

If ABC is a triangle with sides, \(a\), \(b\), and \(c\), then

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.
\]

A is acute

A is obtuse

The Law of Sines can be used when you know:

AAS
ASA
SSA

Law of Cosines

Standard Form

\[
a^2 = b^2 + c^2 - 2bc \cos A
\]

\[
b^2 = a^2 + c^2 - 2ac \cos B
\]

\[
c^2 = a^2 + b^2 - 2ab \cos C
\]

Alternative Form

\[
\cos A = \frac{b^2 + c^2 - a^2}{2bc}
\]

\[
\cos B = \frac{a^2 + c^2 - b^2}{2ac}
\]

\[
\cos C = \frac{a^2 + b^2 - c^2}{2ab}
\]

The Law of Cosines can be used when you know:

SSS
SAS
Area Formulas

Area of an Oblique Triangle – used when you know two sides and their included angle

\[ \text{Area} = \frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B \]

Heron’s Area Formula – used when you know the lengths of all three sides

\[ \text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \]

Where \( s = \frac{a + b + c}{2} \) (this is half the perimeter)